AALBORG UNIVERSITY

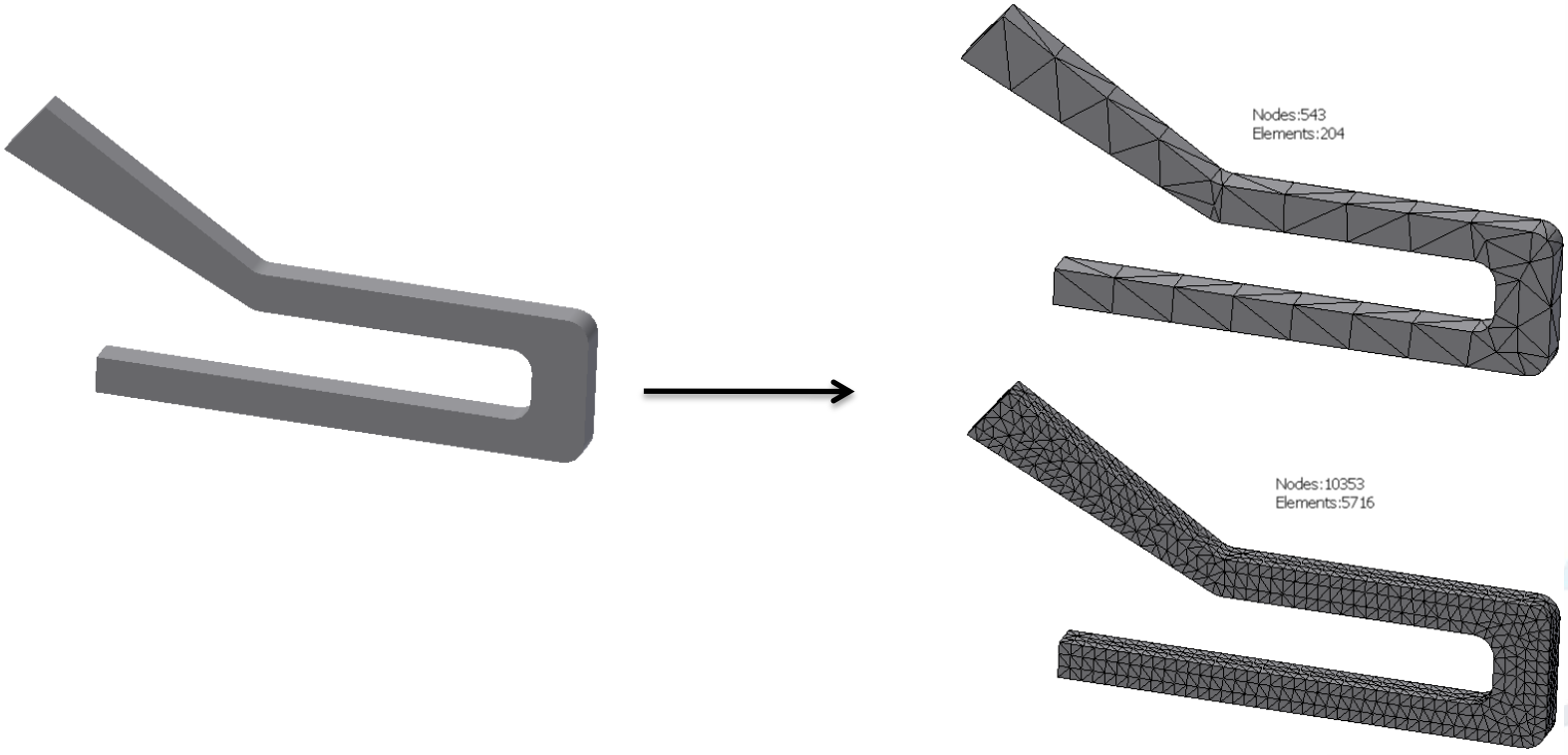
FEM

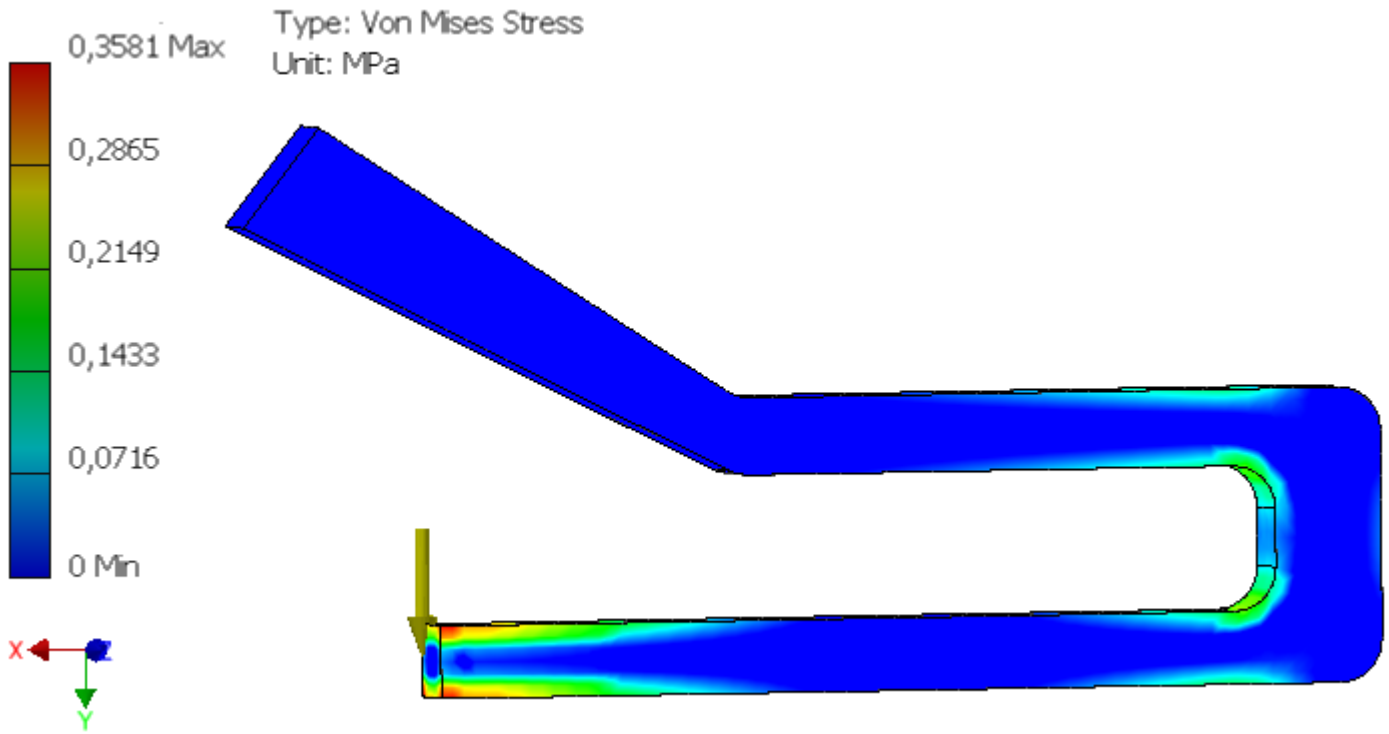
M5

Introduktion til FEM

Søren Heide Lambertsen

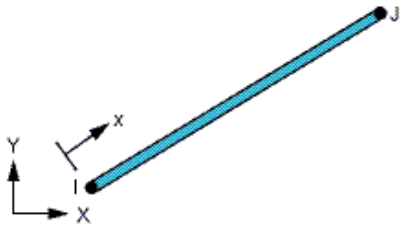
Hvad anvender man FEA til.




0,3581 Max
0,2865
0,2149
0,1433
0,0716
0 Min
Type: Von Mises Stress
Unit: MPa
X
Y

Elementtyper

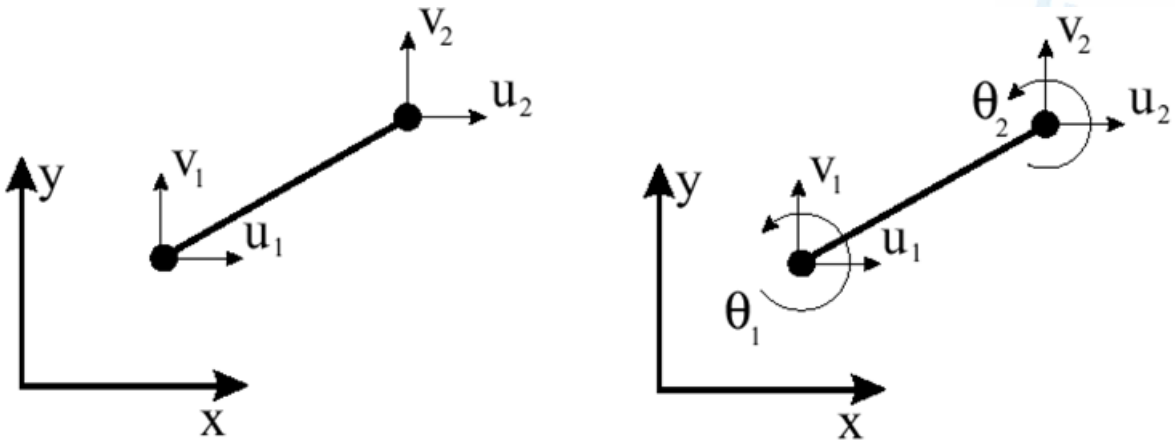
1D



1 frihedsgrad

Elementtyper

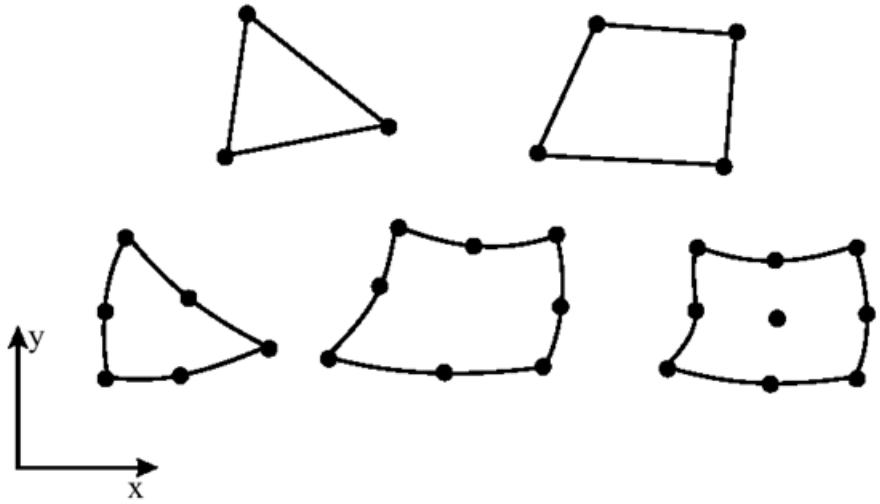
2D



2-3 frihedsgrader

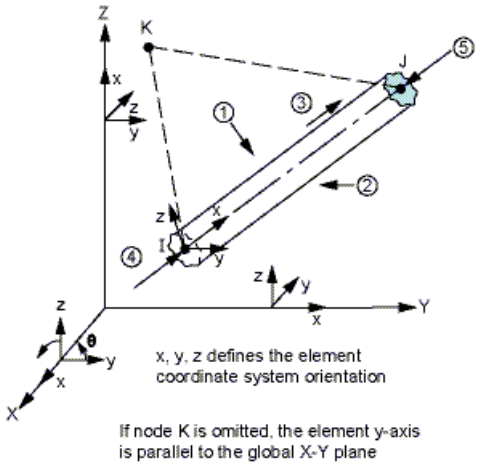
Elementtyper

2D



Elementtyper

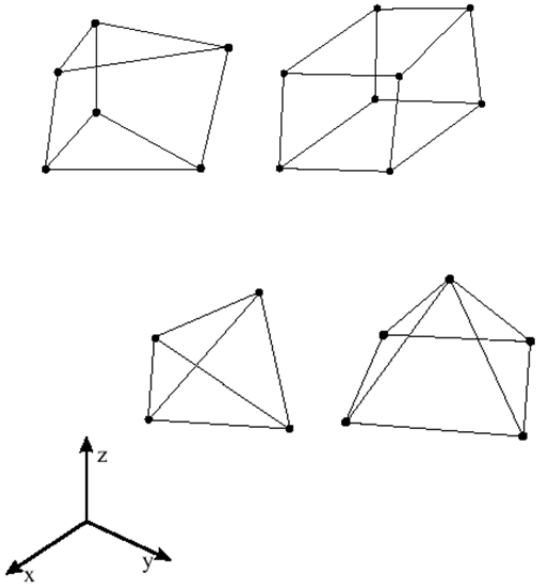
3D



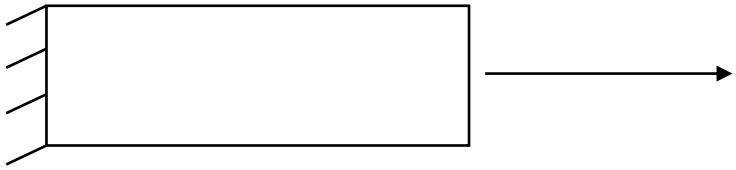
6 frihedsgrader(Bjælke) 3 frihedsgrader for Solid

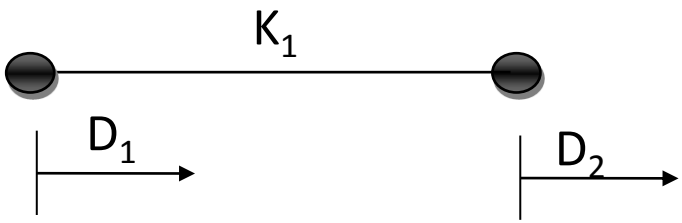
Elementtyper

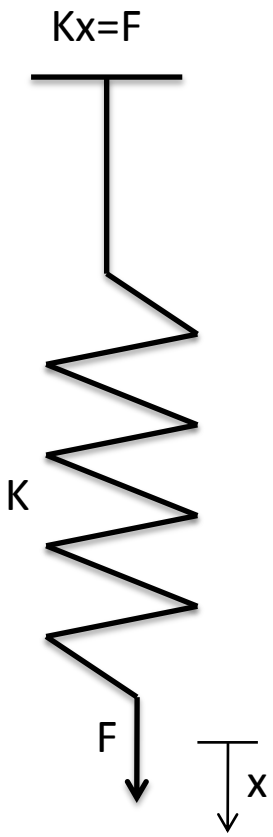
3D



Eksempel 1







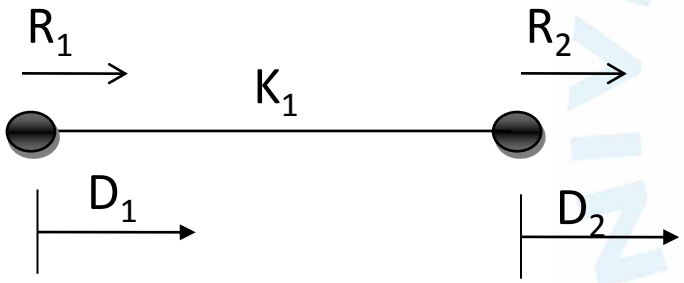
[k]{d}={R}

[Stivhedsmatrice] {Flytninger}= {kraftvektor}

Eksempel 1

$$k = \frac{A \cdot E}{L}$$

[k]{d}={R}



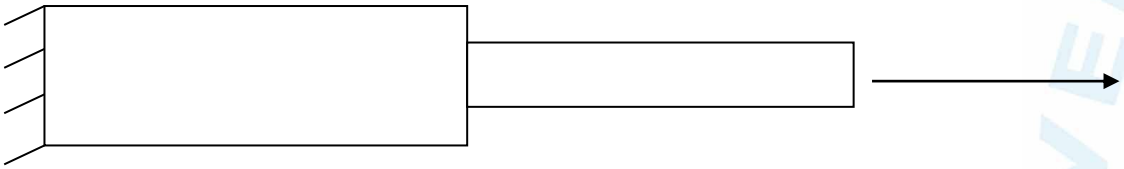
Løsning eksempel 1

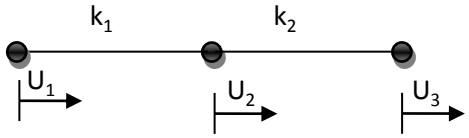
$$[k] \cdot \{D\} = \{F\} \rightarrow \{D\} = [k]^{-1}\{F\}$$

$$k = \frac{A \cdot E}{L}$$

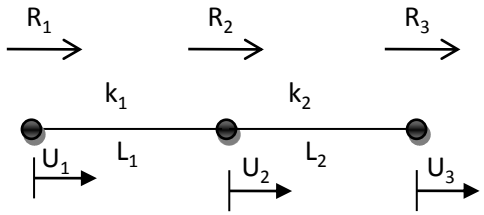
$$\begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

Eksempel 2





Matrice eksempel 2



$$k_1 = \frac{A_1 \cdot E_1}{L_1} \qquad k_2 = \frac{A_2 \cdot E_2}{L_2}$$

- Løsning eksempel 2

$$F_1 = U_1 \cdot k_1 - U_2 \cdot k_1$$
$$F_2 = -U_1 \cdot k_1 - U_3 \cdot k_2 + U_2 \cdot k_2 + U_2 \cdot k_1$$
$$F_3 = U_3 \cdot k_2 - U_2 \cdot k_2$$

$$\begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1 + k_2 & -k_2 \\ k_{31} & -k_2 & k_2 \end{bmatrix} \cdot \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

- Løsning eksempel 2

$$F_2 = 0$$

$$\begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1 + k_2 & -k_2 \\ k_{31} & -k_2 & k_2 \end{bmatrix} \cdot \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

$$\begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ P \end{Bmatrix}$$

$$\begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ P \end{Bmatrix} \Rightarrow \begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix} = \frac{1}{k_1 \cdot k_2} \begin{bmatrix} k_1 & k_2 \\ k_2 & k_1 + k_2 \end{bmatrix} \begin{Bmatrix} 0 \\ P \end{Bmatrix}$$

$$D_1 = \frac{k_2}{k_1 \cdot k_2} \cdot P = \frac{P}{k_1}$$

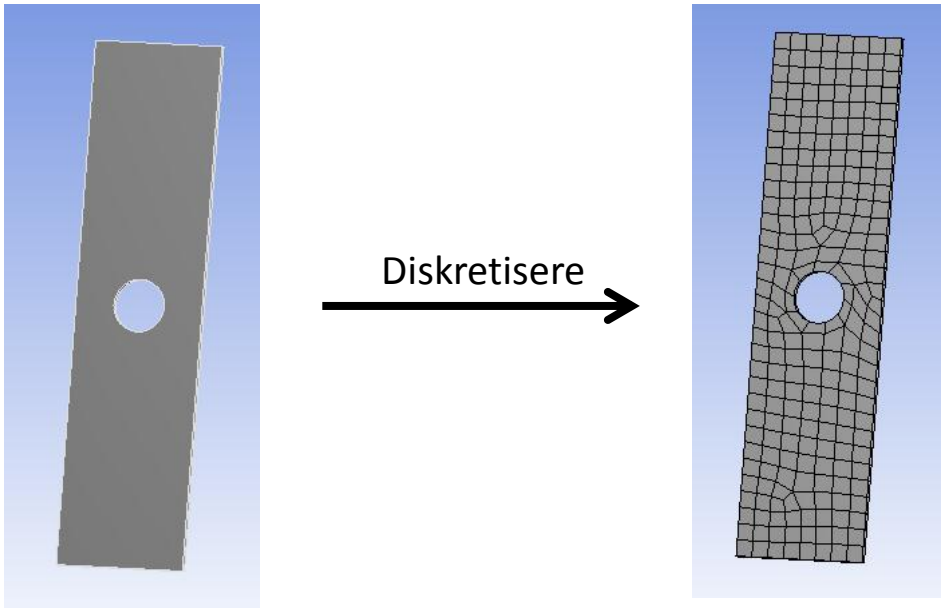
$$D_2 = \frac{k_1 \cdot P}{k_1 \cdot k_2} + \frac{k_2 \cdot P}{k_1 \cdot k_2} = \frac{P}{k_2} + \frac{P}{k_1}$$

$$D_1 = \frac{P}{k_1} = \frac{P \cdot L_1}{A_1 \cdot E_1}$$

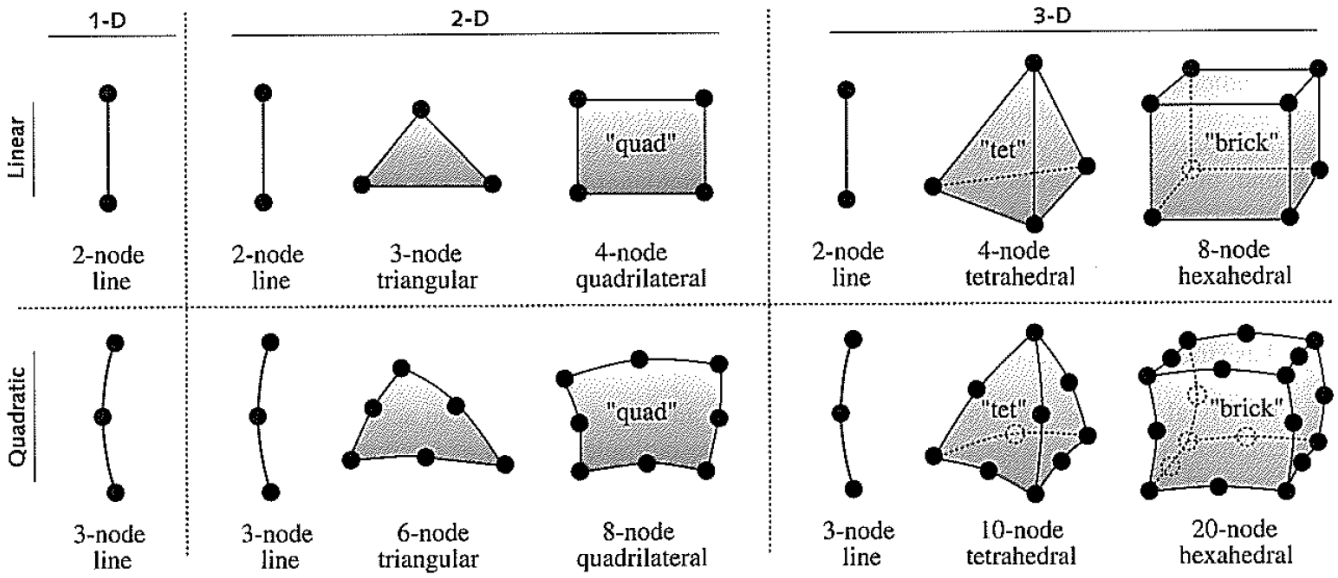
$$D_2 = \frac{P \cdot L_2}{A_2 \cdot E_2} + \frac{P \cdot L_1}{A_1 \cdot E_1}$$

Pause

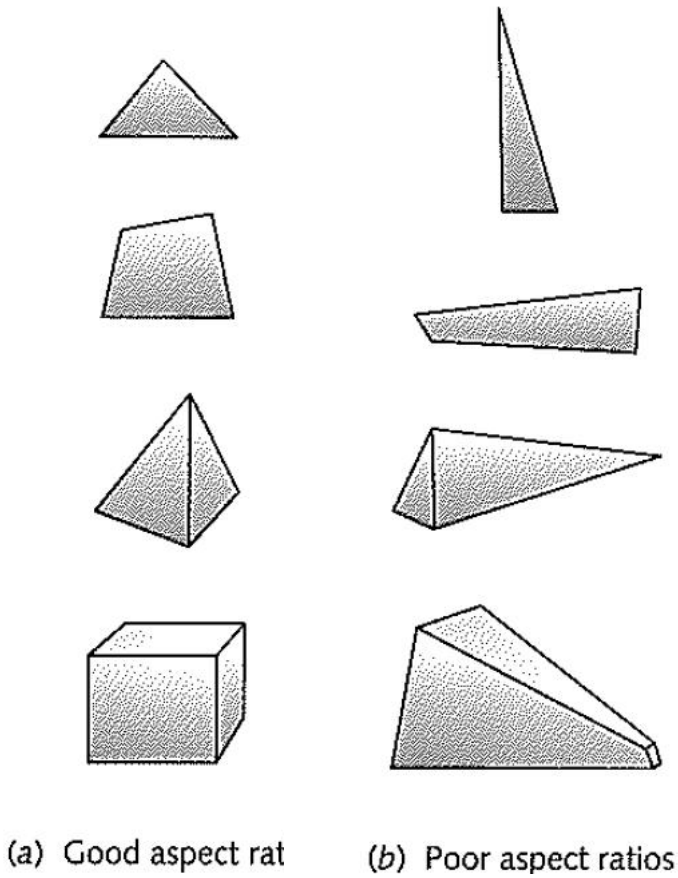
Diskretisering af model



Elementtyper

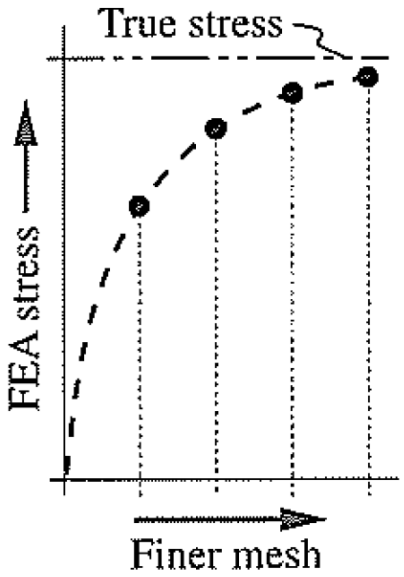


Mesh



(a) Good aspect rat (b) Poor aspect ratios

Convergense



Bjælkeelement.

DOF = ?

Stivhedsmatricen

6X6 matrice

$$\begin{bmatrix} [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \end{bmatrix} \cdot \begin{Bmatrix} ? \\ ? \\ ? \\ ? \\ ? \\ ? \end{Bmatrix} = \begin{Bmatrix} ? \\ ? \\ ? \\ ? \\ ? \\ ? \end{Bmatrix}$$

Stivhedsmatricen

$$\begin{bmatrix} [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} R_{u1} \\ R_{v1} \\ R_{\theta 1} \\ R_{u2} \\ R_{v2} \\ R_{\theta 2} \end{Bmatrix}$$

Stivhedsmatricen

$$\begin{bmatrix} [?] & [?] & [?] & [?] & [?] & [?] \\ [?] & [?] & [?] & [\ ] & [\ ] & [\ ] \\ [?] & [?] & [?] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} R_{u1} \\ R_{v1} \\ R_{\theta 1} \\ R_{u2} \\ R_{v2} \\ R_{\theta 2} \end{Bmatrix}$$

Stivhedsmatricen

$$\begin{bmatrix} [EA/L] & [0] & [0] & [?] & [?] & [?] \\ [?] & [?] & [?] & [\ ] & [\ ] & [\ ] \\ [?] & [?] & [?] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} R_{u1} \\ R_{v1} \\ R_{\theta 1} \\ R_{u2} \\ R_{v2} \\ R_{\theta 2} \end{Bmatrix}$$

Stivhedsmatricen

$$\begin{bmatrix} [EA/L] & [0] & [0] & [-EA/L] & [0] & [0] \\ [?] & [?] & [?] & [\ ] & [\ ] & [\ ] \\ [?] & [?] & [?] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} R_{u1} \\ R_{v1} \\ R_{\theta 1} \\ R_{u2} \\ R_{v2} \\ R_{\theta 2} \end{Bmatrix}$$

Stivhedsmatricen

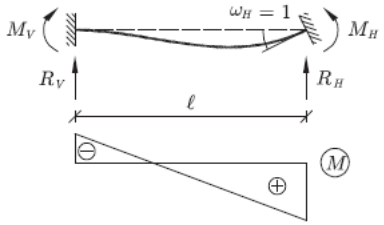
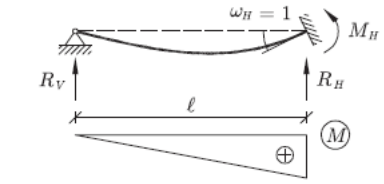
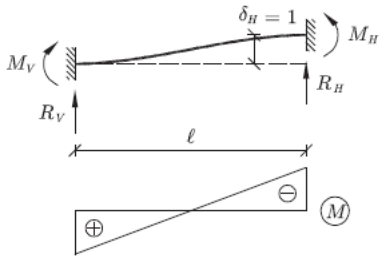
http://people.civil.aau.dk/~lda/Notes/

Deformationsmetoden for Rammekonstruktioner

Side 72

Stivhedsmatricen

Tabel 1: Bjælker med forskellige foreskrevne drejninger og flytninger

	$M_V = -2\frac{EI}{\ell}$ $M_H = 4\frac{EI}{\ell}$ $R_V = 6\frac{EI}{\ell^2}$ $R_H = -6\frac{EI}{\ell^2}$
	$M_H = 3\frac{EI}{\ell}$ $R_V = 3\frac{EI}{\ell^2}$ $R_H = -3\frac{EI}{\ell^2}$
	$M_V = 6\frac{EI}{\ell^2}$ $M_H = -6\frac{EI}{\ell^2}$ $R_V = -12\frac{EI}{\ell^3}$ $R_H = 12\frac{EI}{\ell^3}$

Stivhedsmatricen

$$\begin{bmatrix} [EA/L] & [0] & [0] & [-EA/L] & [0] & [0] \\ [?] & [?] & [?] & [\ ] & [\ ] & [\ ] \\ [?] & [?] & [?] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} R_{u1} \\ R_{v1} \\ R_{\theta 1} \\ R_{u2} \\ R_{v2} \\ R_{\theta 2} \end{Bmatrix}$$

Stivhedsmatricen

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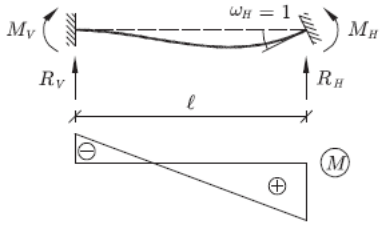
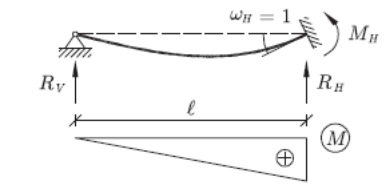
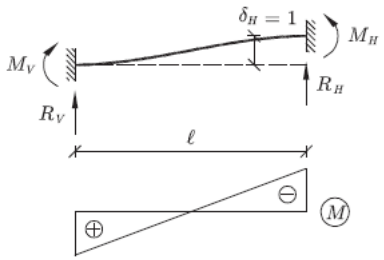
	$M_V = -2\frac{EI}{\ell}$ $M_H = 4\frac{EI}{\ell}$ $R_V = 6\frac{EI}{\ell^2}$ $R_H = -6\frac{EI}{\ell^2}$
	$M_H = 3\frac{EI}{\ell}$ $R_V = 3\frac{EI}{\ell^2}$ $R_H = -3\frac{EI}{\ell^2}$
	$M_V = 6\frac{EI}{\ell^2}$ $M_H = -6\frac{EI}{\ell^2}$ $R_V = -12\frac{EI}{\ell^3}$ $R_H = 12\frac{EI}{\ell^3}$

Stivhedsmatricen

$$\begin{bmatrix} [EA/L] & [0] & [0] & [-EA/L] & [0] & [0] \\ [0] & [12EI/L^3] & [?] & [\ ] & [\ ] & [\ ] \\ [?] & [?] & [?] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} R_{u1} \\ R_{v1} \\ R_{\theta 1} \\ R_{u2} \\ R_{v2} \\ R_{\theta 2} \end{Bmatrix}$$

Stivhedsmatricen

$$\begin{bmatrix} [EA/L] & [0] & [0] & [-EA/L] & [0] & [0] \\ [0] & [12EI/L^3] & [?] & [\ ] & [\ ] & [\ ] \\ [?] & [?] & [?] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} R_{u1} \\ R_{v1} \\ R_{\theta 1} \\ R_{u2} \\ R_{v2} \\ R_{\theta 2} \end{Bmatrix}$$

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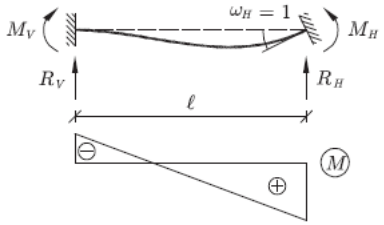
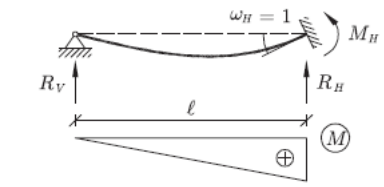
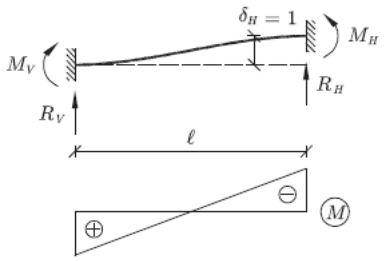
	$M_V = -2\frac{EI}{\ell}$ $M_H = 4\frac{EI}{\ell}$ $R_V = 6\frac{EI}{\ell^2}$ $R_H = -6\frac{EI}{\ell^2}$
	$M_H = 3\frac{EI}{\ell}$ $R_V = 3\frac{EI}{\ell^2}$ $R_H = -3\frac{EI}{\ell^2}$
	$M_V = 6\frac{EI}{\ell^2}$ $M_H = -6\frac{EI}{\ell^2}$ $R_V = -12\frac{EI}{\ell^3}$ $R_H = 12\frac{EI}{\ell^3}$

Stivhedsmatricen

$$\begin{bmatrix} [EA/L] & [0] & [0] & [-EA/L] & [0] & [0] \\ [0] & [12EI/L^3] & [6EA/L] & [\ ] & [\ ] & [\ ] \\ [?] & [?] & [?] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} R_{u1} \\ R_{v1} \\ R_{\theta 1} \\ R_{u2} \\ R_{v2} \\ R_{\theta 2} \end{Bmatrix}$$

Stivhedsmatricen

$$\begin{bmatrix} [EA/L] & [0] & [0] & [-EA/L] & [0] & [0] \\ [0] & [12EI/L^3] & [6EA/L] & [\ ] & [\ ] & [\ ] \\ 0 & [?] & [?] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} R_{u1} \\ R_{v1} \\ R_{\theta 1} \\ R_{u2} \\ R_{v2} \\ R_{\theta 2} \end{Bmatrix}$$

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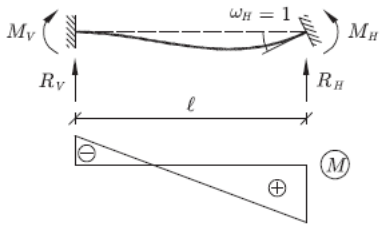
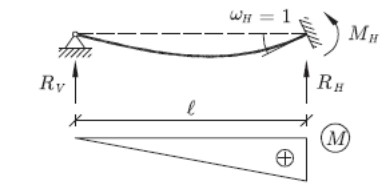
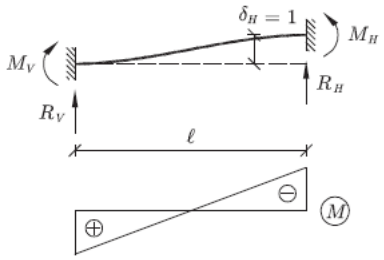
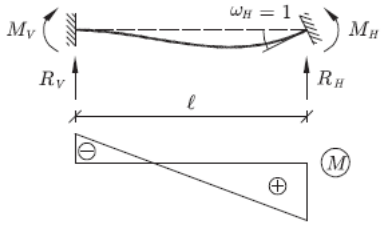
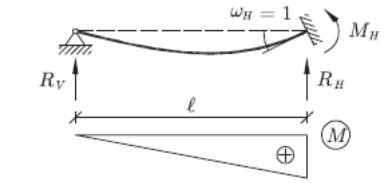
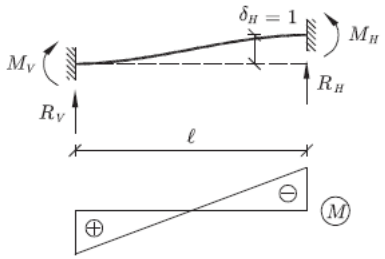
	$M_V = -2\frac{EI}{\ell}$ $M_H = 4\frac{EI}{\ell}$ $R_V = 6\frac{EI}{\ell^2}$ $R_H = -6\frac{EI}{\ell^2}$
	$M_H = 3\frac{EI}{\ell}$ $R_V = 3\frac{EI}{\ell^2}$ $R_H = -3\frac{EI}{\ell^2}$
	$M_V = 6\frac{EI}{\ell^2}$ $M_H = -6\frac{EI}{\ell^2}$ $R_V = -12\frac{EI}{\ell^3}$ $R_H = 12\frac{EI}{\ell^3}$

Stivhedsmatricen

$$\begin{bmatrix} [EA/L] & [0] & [0] & [-EA/L] & [0] & [0] \\ [0] & [12EI/L^3] & [6EA/L] & [\ ] & [\ ] & [\ ] \\ 0 & [-6EA/L^2] & [?] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} R_{u1} \\ R_{v1} \\ R_{\theta 1} \\ R_{u2} \\ R_{v2} \\ R_{\theta 2} \end{Bmatrix}$$

Stivhedsmatricen

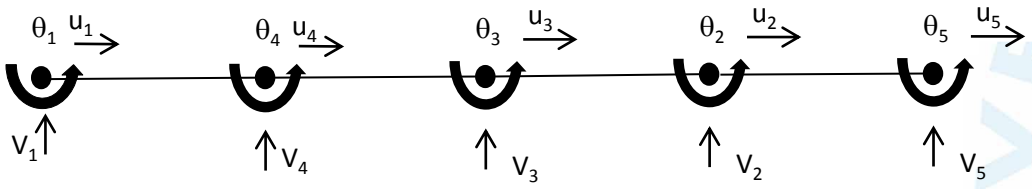
Tabel 1: Bjælker med forskellige foreskrevne drejninger og flytninger

	$M_V = -2\frac{EI}{\ell}$ $M_H = 4\frac{EI}{\ell}$ $R_V = 6\frac{EI}{\ell^2}$ $R_H = -6\frac{EI}{\ell^2}$
	$M_H = 3\frac{EI}{\ell}$ $R_V = 3\frac{EI}{\ell^2}$ $R_H = -3\frac{EI}{\ell^2}$
	$M_V = 6\frac{EI}{\ell^2}$ $M_H = -6\frac{EI}{\ell^2}$ $R_V = -12\frac{EI}{\ell^3}$ $R_H = 12\frac{EI}{\ell^3}$

Stivhedsmatricen

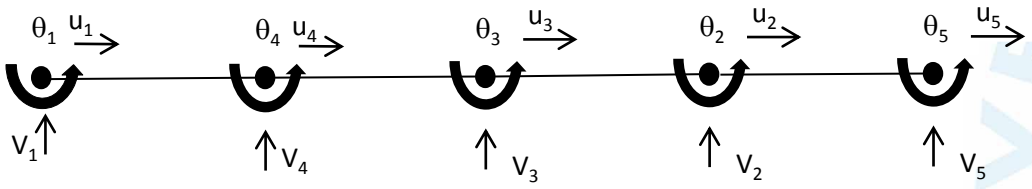
$$\begin{bmatrix} [EA/L] & [0] & [0] & [-EA/L] & [0] & [0] \\ [0] & [12EI/L^3] & [6EA/L] & [\ ] & [\ ] & [\ ] \\ 0 & [6EA/L^2] & [4EA/L] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} R_{u1} \\ R_{v1} \\ R_{\theta 1} \\ R_{u2} \\ R_{v2} \\ R_{\theta 2} \end{Bmatrix}$$

Stivhedsmatricen



Frihedsgrader og matrice

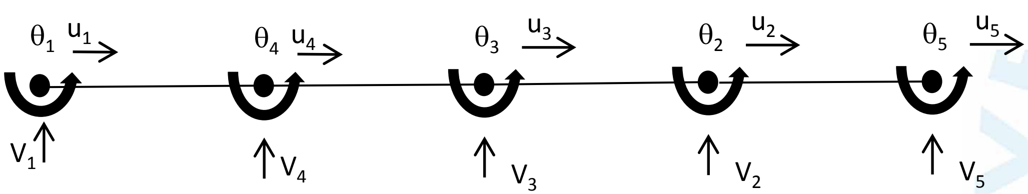
Stivhedsmatricen



3 frihedsgrader i hvert punkt

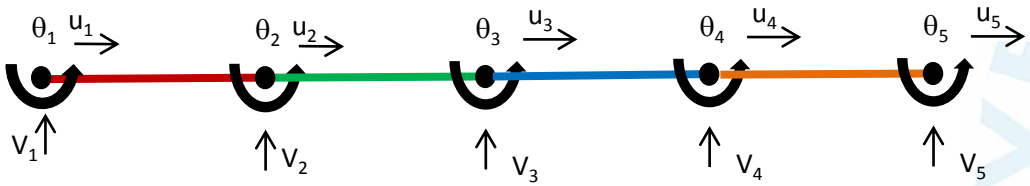
15X15 matrice

Stivhedsmatricen



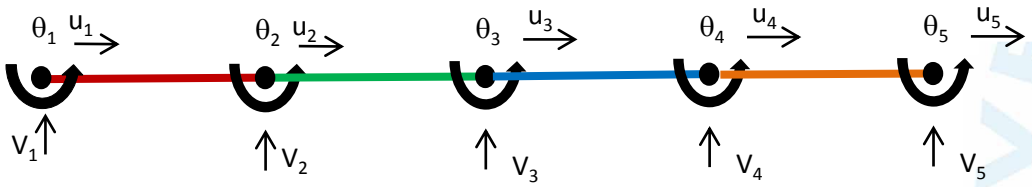
$$\begin{bmatrix} [\quad] & [\quad] & [\quad] & [\quad] & [\quad] \\ [\quad] & [\quad] & [\quad] & [\quad] & [\quad] \\ [\quad] & [\quad] & [\quad] & [\quad] & [\quad] \\ [\quad] & [\quad] & [\quad] & [\quad] & [\quad] \\ [\quad] & [\quad] & [\quad] & [\quad] & [\quad] \end{bmatrix}$$

Stivhedsmatricen



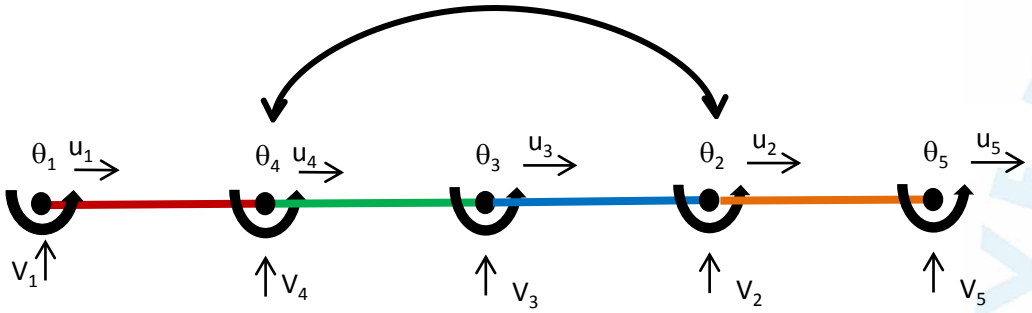
$$\begin{bmatrix} [X] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \end{bmatrix}$$

Stivhedsmatricen



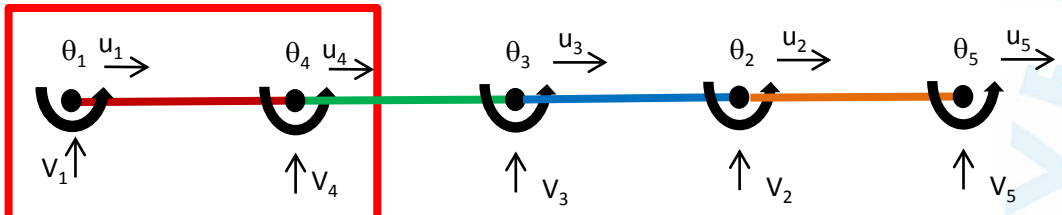
$$\begin{bmatrix} [X] & [X] & [\ ] & [\ ] & [\ ] \\ [X] & [XX] & [X] & [\ ] & [\ ] \\ [\ ] & [X] & [XX] & [X] & [\ ] \\ [\ ] & [\ ] & [X] & [XX] & [X] \\ [\ ] & [\ ] & [\ ] & [X] & [X] \end{bmatrix}$$

Stivhedsmatricen



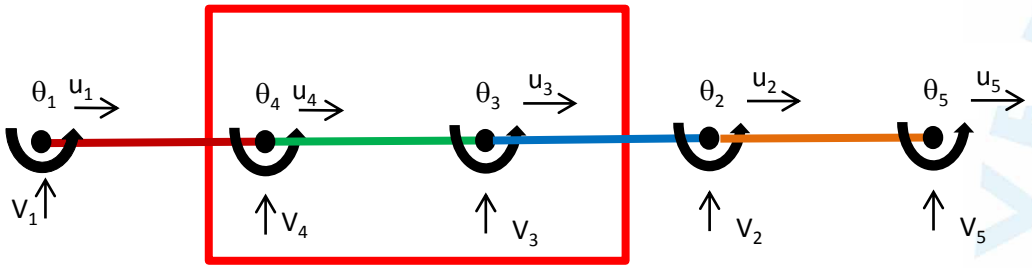
Nyt nummersystem.

Stivhedsmatricen



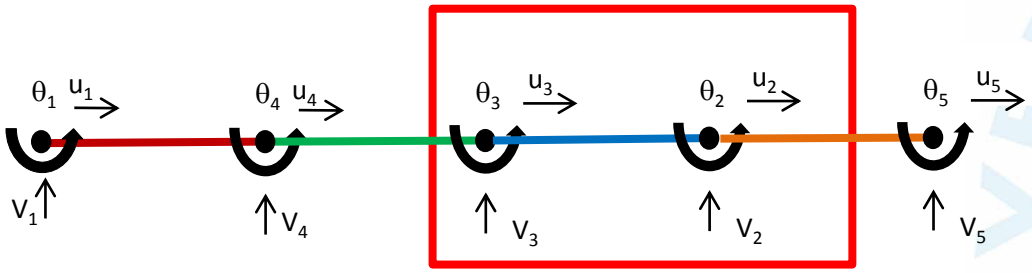
$$\begin{bmatrix} [X] & [\ ] & [\ ] & [X] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [X] & [\ ] & [\ ] & [X] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \end{bmatrix}$$

Stivhedsmatricen



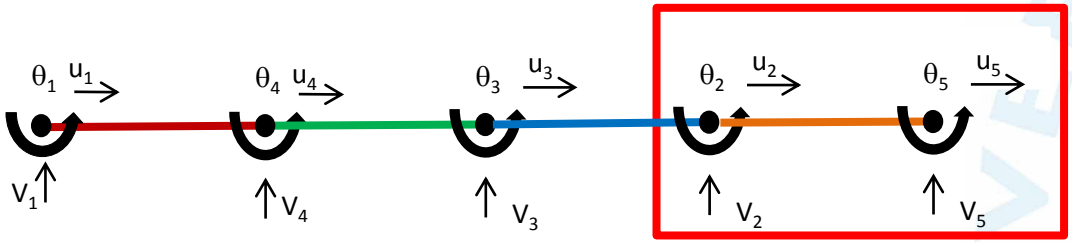
$$\begin{bmatrix} [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [X] & [X] & [\ ] \\ [\ ] & [\ ] & [X] & [X] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \end{bmatrix}$$

Stivhedsmatricen



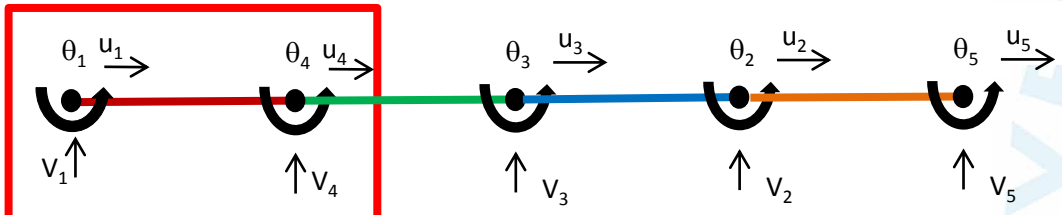
$$\begin{bmatrix} [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [X] & [X] & [\ ] & [\ ] \\ [\ ] & [X] & [X] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \end{bmatrix}$$

Stivhedsmatricen



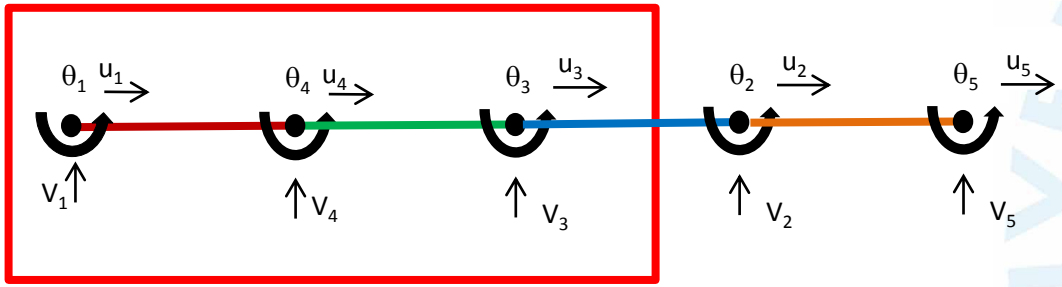
$$\begin{bmatrix} [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [X] & [\ ] & [\ ] & [X] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [X] & [\ ] & [\ ] & [X] \end{bmatrix}$$

Stivhedsmatricen



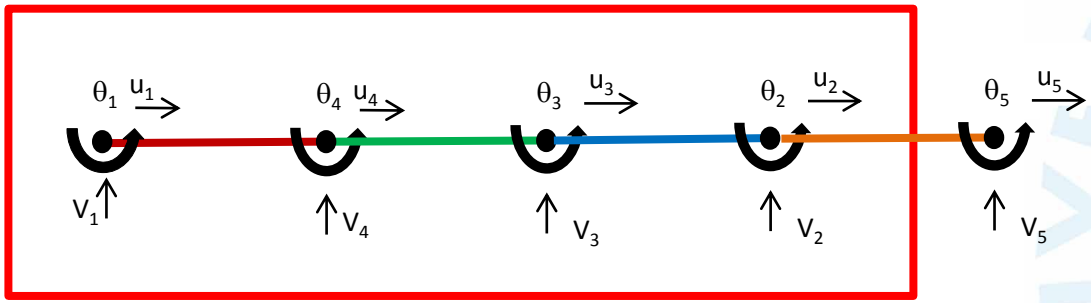
$$\begin{bmatrix} [X] & [\ ] & [\ ] & [X] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [X] & [\ ] & [\ ] & [X] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \end{bmatrix}$$

Stivhedsmatricen



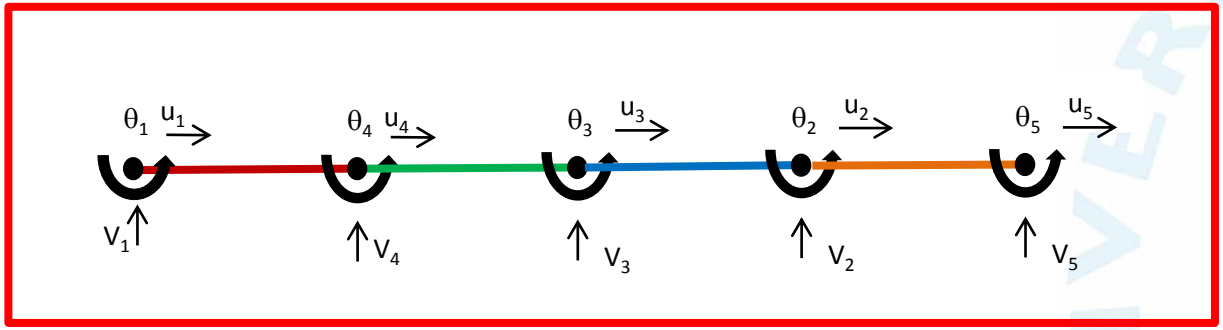
$$\begin{bmatrix} [X] & [\ ] & [\ ] & [X] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \\ [\ ] & [\ ] & [X] & [X] & [\ ] \\ [X] & [\ ] & [X] & [XX] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \end{bmatrix}$$

Stivhedsmatricen



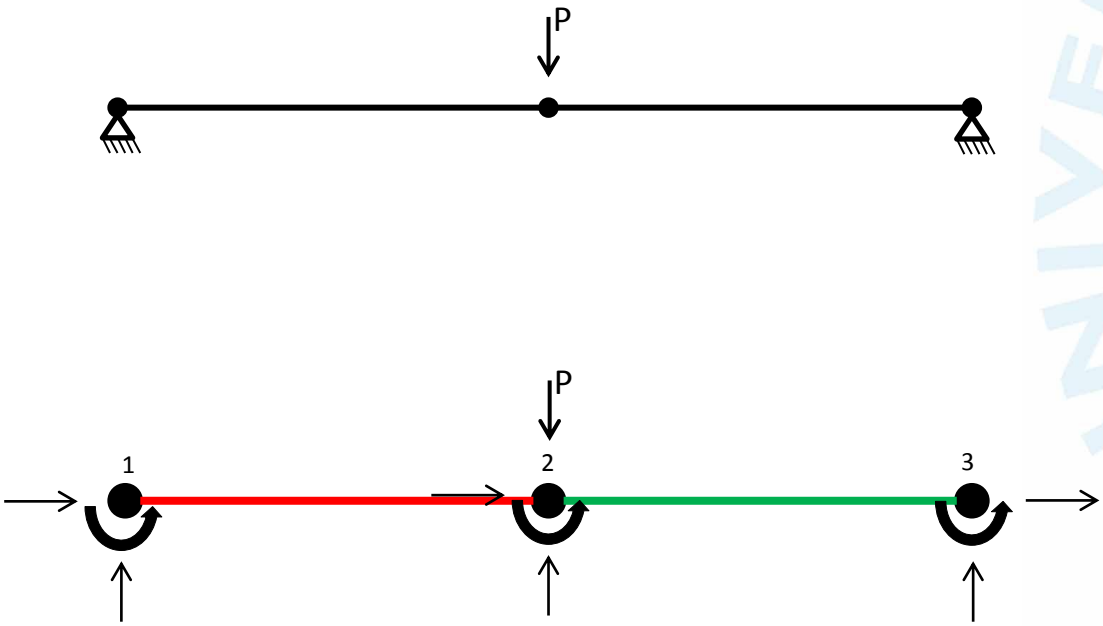
$$\begin{bmatrix} [X] & [\ ] & [\ ] & [X] & [\ ] \\ [\ ] & [X] & [X] & [\ ] & [\ ] \\ [\ ] & [X] & [XX] & [X] & [\ ] \\ [X] & [\ ] & [X] & [XX] & [\ ] \\ [\ ] & [\ ] & [\ ] & [\ ] & [\ ] \end{bmatrix}$$

Stivhedsmatricen

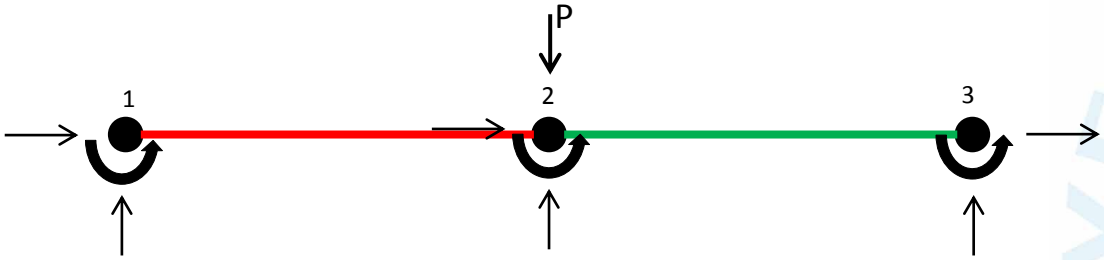


$$\begin{bmatrix} [X] & [\ ] & [\ ] & [X] & [\ ] \\ [\ ] & [XX] & [X] & [\ ] & [X] \\ [\ ] & [X] & [XX] & [X] & [\ ] \\ [X] & [\ ] & [X] & [XX] & [\ ] \\ [\ ] & [X] & [\ ] & [\ ] & [X] \end{bmatrix}$$

Eksempel

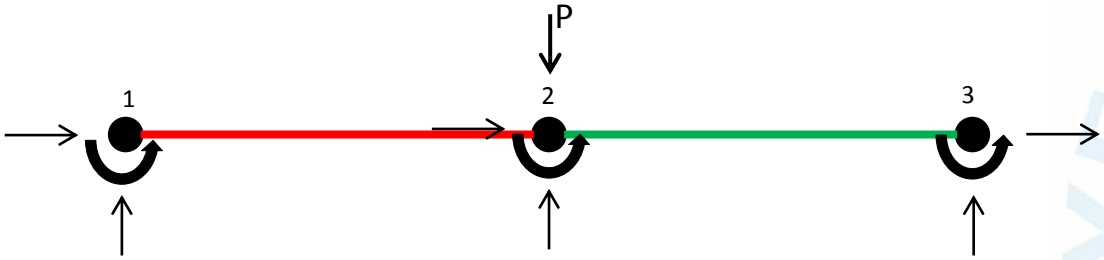


Eksempel



$$\{D\} = \begin{bmatrix} 0 \\ 0 \\ ? \\ \hline ? \\ ? \\ ? \\ \hline 0 \\ 0 \\ ? \end{bmatrix}$$

Eksempel



$$\{R\} = \begin{bmatrix} R_{1N} \\ R_{1V} \\ \hline 0 \\ 0 \\ -P \\ 0 \\ \hline R_{3N} \\ R_{3V} \\ 0 \end{bmatrix}$$

Eksempel

$$\begin{bmatrix} [X][X][X] & [X] & [X] & [X] & [0] & [0] & [0] \\ [X][X][X] & [X] & [X] & [X] & [0] & [0] & [0] \\ [X][X][X] & [X] & [X] & [X] & [0] & [0] & [0] \\ \hline [X][X][X] & [XX] & [XX] & [XX] & [X] & [X] & [X] \\ [X][X][X] & [XX] & [XX] & [XX] & [X] & [X] & [X] \\ [X][X][X] & [XX] & [XX] & [XX] & [X] & [X] & [X] \\ \hline [0][0][0] & [X] & [X] & [X] & [X] & [X] & [X] \\ [0][0][0] & [X] & [X] & [X] & [X] & [X] & [X] \\ [0][0][0] & [X] & [X] & [X] & [X] & [X] & [X] \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ ? \\ ? \\ ? \\ ? \\ 0 \\ 0 \\ ? \end{bmatrix} = \begin{bmatrix} R_{1N} \\ R_{1V} \\ 0 \\ 0 \\ -P \\ 0 \\ R_{3N} \\ R_{3V} \\ 0 \end{bmatrix}$$

Eksempel

$$\left[\begin{array}{ccccc|ccc}
[X] & [X] & [X] & [XX] & [XX] & [XX] & [X] & [X] & [X] \\
[X] & [X] & [X] & [XX] & [XX] & [XX] & [X] & [X] & [X] \\
[X] & [X] & [X] & [XX] & [XX] & [XX] & [X] & [X] & [X] \\
[X] & [X] & [X] & [X] & [X] & [X] & [0] & [0] & [0] \\
[0] & [0] & [0] & [XX] & [XX] & [XX] & [X] & [X] & [X] \\
\hline
[X] & [X] & [X] & [X] & [X] & [X] & [0] & [0] & [0] \\
[X] & [X] & [X] & [X] & [X] & [X] & [0] & [0] & [0] \\
[0] & [0] & [0] & [X] & [X] & [X] & [X] & [X] & [X] \\
[0] & [0] & [0] & [X] & [X] & [X] & [X] & [X] & [X]
\end{array}\right] \cdot \left[\begin{array}{c} ? \\ ? \\ ? \\ ? \\ ? \\ \hline 0 \\ 0 \\ 0 \\ 0 \end{array}\right] = \left[\begin{array}{c} 0 \\ -P \\ 0 \\ 0 \\ 0 \\ \hline R_{1N} \\ R_{1V} \\ R_{3N} \\ R_{3V} \end{array}\right]$$

Eksempel


$[k_{reduceret}]$
$[k_{12}]$
$\begin{bmatrix} X & X & X & XX & XX & XX & X & X & X \\ X & X & X & XX & XX & XX & X & X & X \\ X & X & X & XX & XX & XX & X & X & X \\ X & X & X & X & X & X & 0 & 0 & 0 \\ 0 & 0 & 0 & XX & XX & XX & X & X & X \\ X & X & X & X & X & X & 0 & 0 & 0 \\ X & X & X & X & X & X & 0 & 0 & 0 \\ 0 & 0 & 0 & X & X & X & X & X & X \\ 0 & 0 & 0 & X & X & X & X & X & X \end{bmatrix} \cdot \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -P \\ 0 \\ 0 \\ 0 \\ R_{1N} \\ R_{1V} \\ R_{3N} \\ R_{3V} \end{bmatrix}$

Eksempel

$$[k_{reduceret}]\{D\} = \{R\}$$

$$\{D\} = \{R\}[k_{reduceret}]^{-1}$$

Reaktioner bliver da:

$$\{R_{reaktioner}\} = [k_{12}]\{D\}$$

AALBORG UNIVERSITY

FEM

BM5 kursusgang

Introduktion til FEM (Modal analyse)

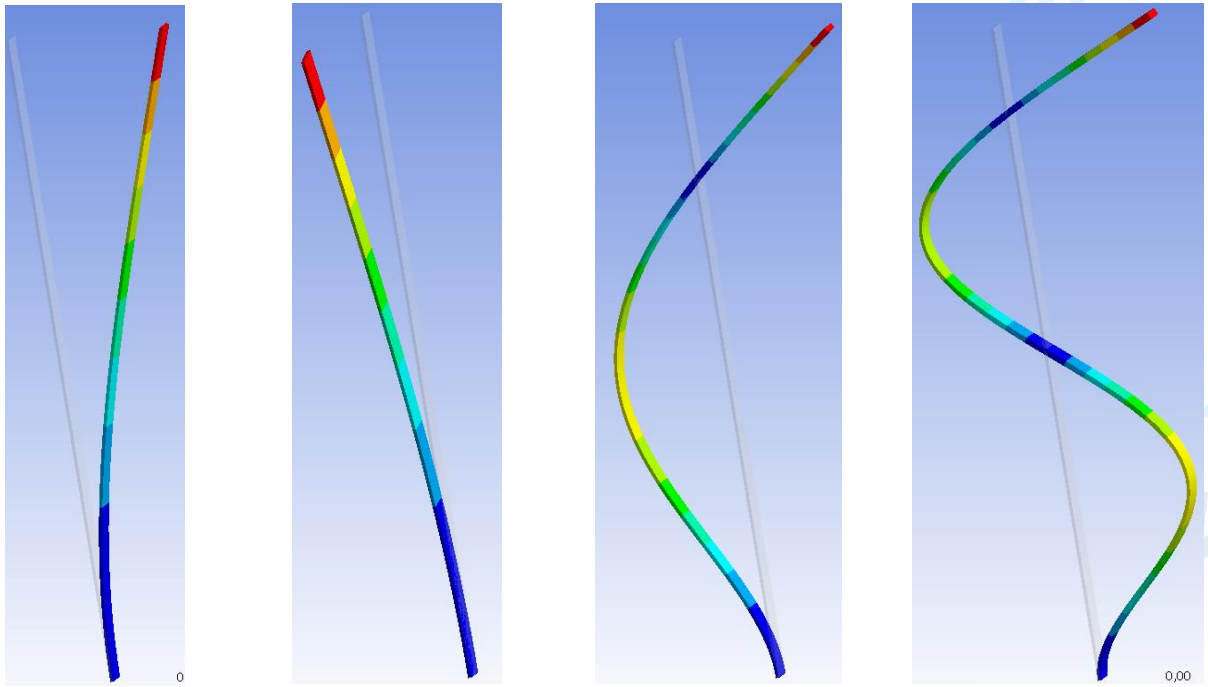
Søren Heide Lambertsen

I dag

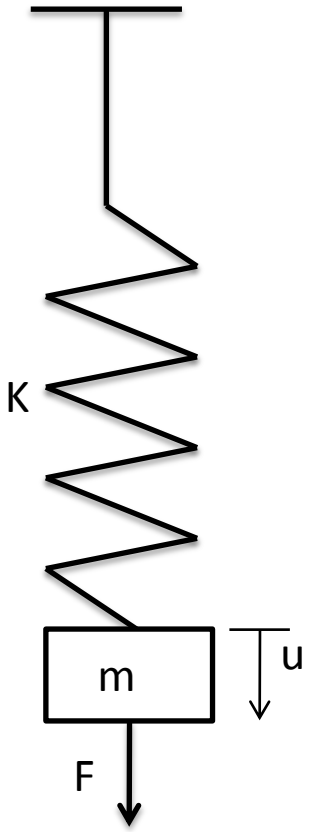
1. Hvad er modal analyse
2. Egenfrekvens,Dæmpning
3. Beregning´s grundlaget for modal analyse
4. Eksempel 1
5. Opsætning af modal analyse i Ansys workbench
6. Opsætning af Prestress modal analyse i Ansys Workbench

Modal analyse

Beregningsmetoden giver mulighed for at bestemme egenfrekvenser for en struktur.

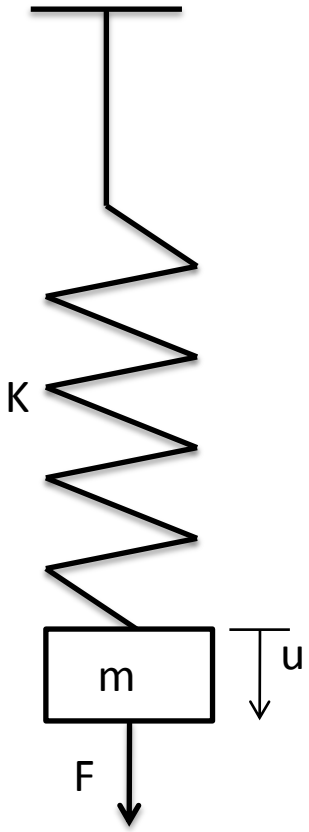


Fjeder/masse system



$$u = u(t)$$

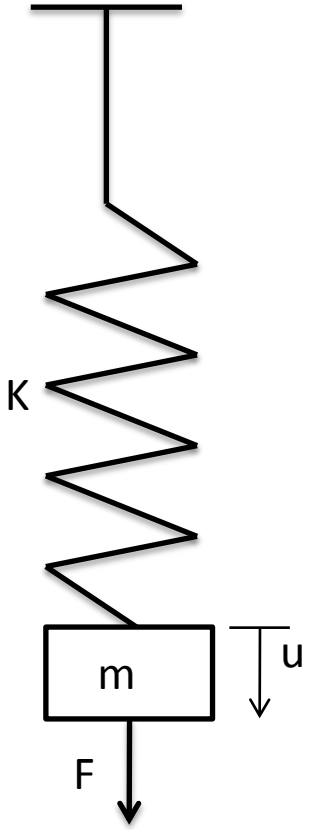
Fjeder/masse system



$$u = u(t)$$

$$\dot{u} = \frac{du}{dt}$$

Fjeder/masse system

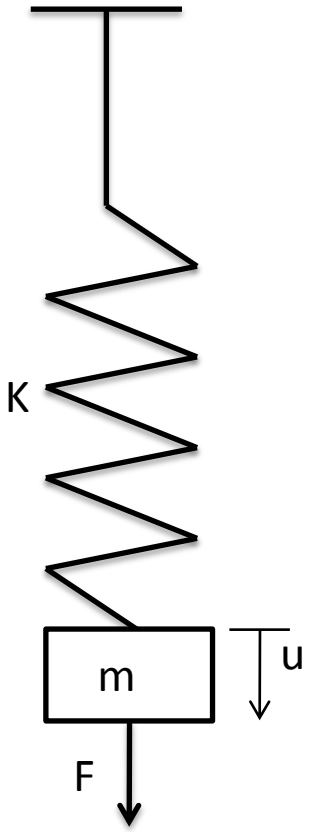


$$u = u(t)$$

$$\dot{u} = \frac{du}{dt}$$

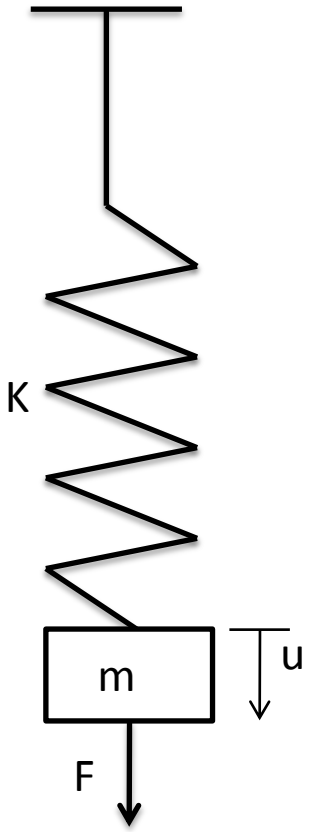
$$a = \ddot{u} = \frac{du^2}{d^2t}$$

Fjeder/masse system



$$u = u\sin(\omega t)$$

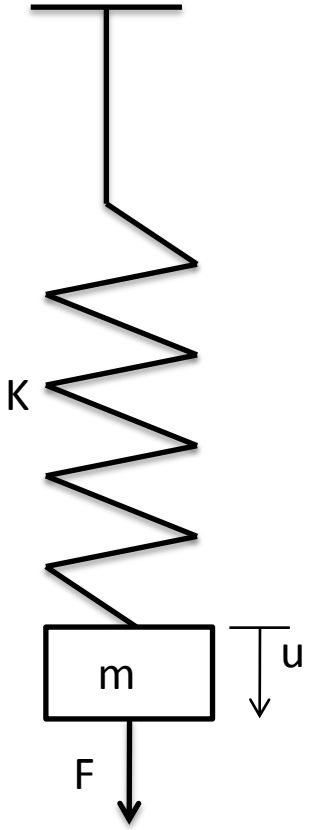
Fjeder/masse system



$$u = u\,\sin(\omega t)$$

$$\dot{u} = u\omega\,\cos(\omega t)$$

Fjeder/masse system

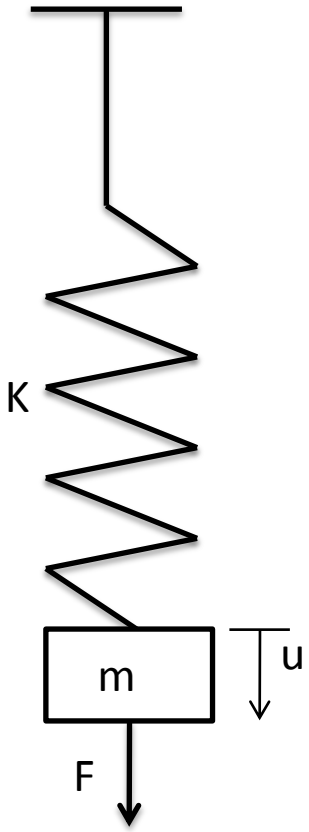


$$u = u\,\sin(\omega t)$$

$$\dot{u} = u\omega\,\cos(\omega t)$$

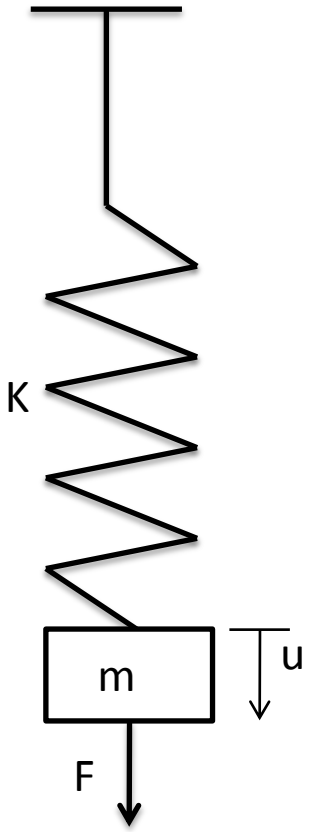
$$a = \ddot{u} = -u\omega^2 \sin(\omega t)$$

Fjeder/masse system



$$f = ma \Rightarrow f = m\ddot{u}$$

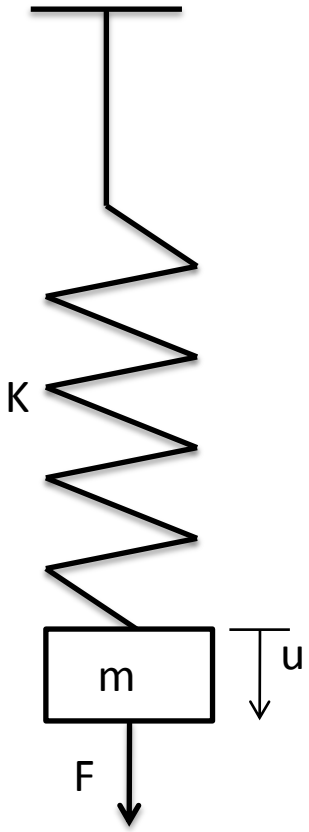
Fjeder/masse system



$$f = ma \Rightarrow f = m\ddot{u}$$

$$f = m\ddot{u} \Rightarrow f - \mathrm{ku} - c\dot{u} = \mathrm{m}\ddot{u}$$

Fjeder/masse system

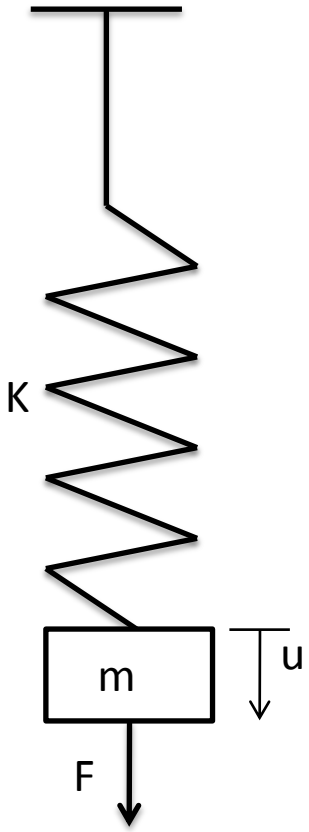


$$f = ma \Rightarrow f = m\ddot{u}$$

$$f = m\ddot{u} \Rightarrow f - \mathrm{ku} - c\dot{u} = \mathrm{m}\ddot{u}$$

$$f = \mathrm{m}\ddot{u} + c\dot{u} + ku$$

Fjeder/masse system

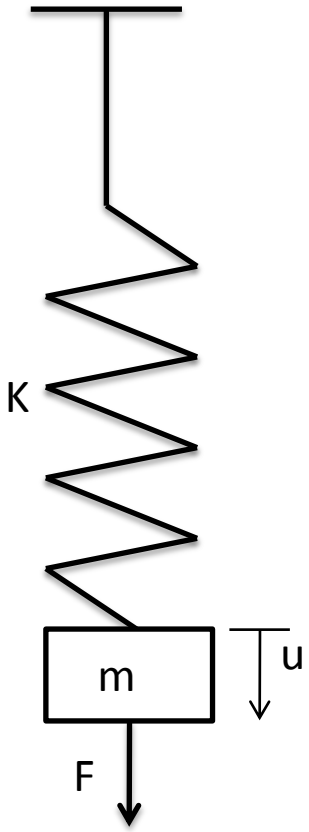


$$f = ma \Rightarrow f = m\ddot{u}$$

$$f = m\ddot{u} \Rightarrow f - \mathrm{ku} - c\dot{u} = \mathrm{m}\ddot{u}$$

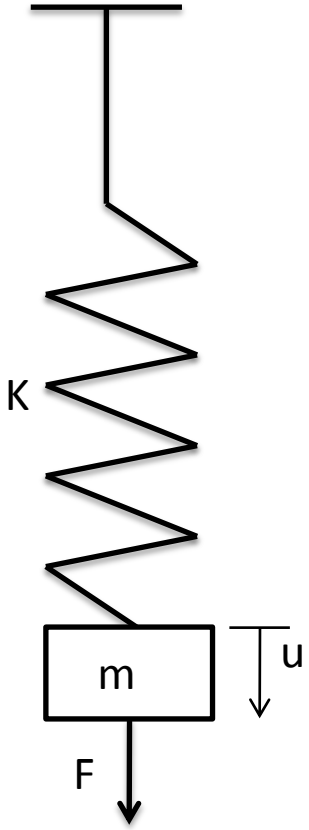
$$f = \mathrm{m}\ddot{u} + c\dot{u} + ku$$

Fjeder/masse system



$$c = 0 \quad f = 0$$

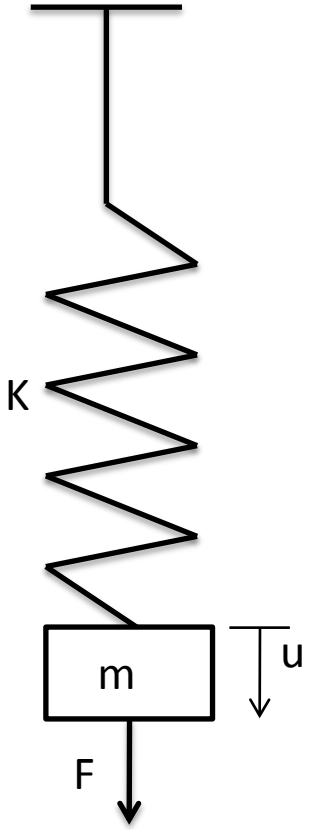
Fjeder/masse system



$$c = 0 \quad f = 0$$

$$\cancel{f} = \mathrm{m}\ddot{u} + \cancel{c\dot{u}} + ku$$

Fjeder/masse system



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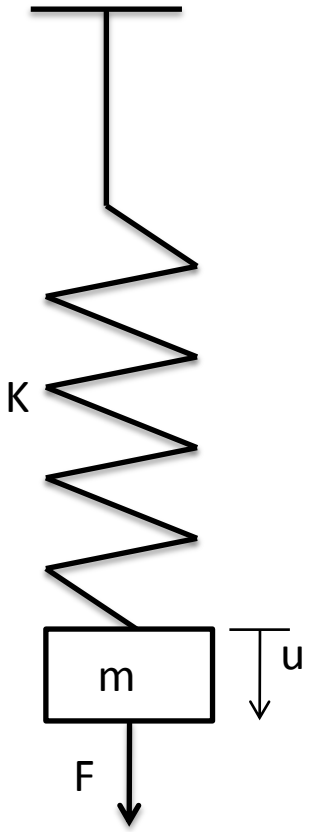

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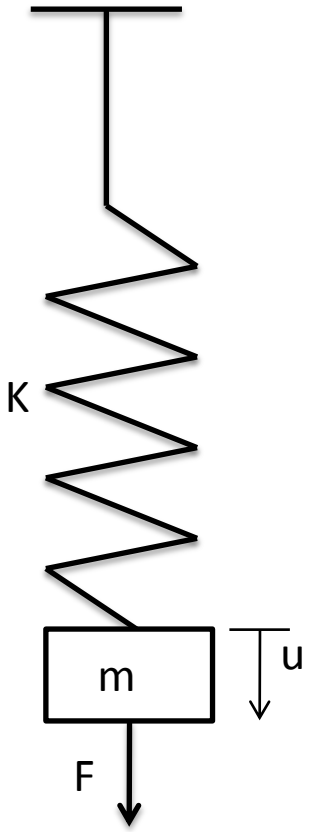
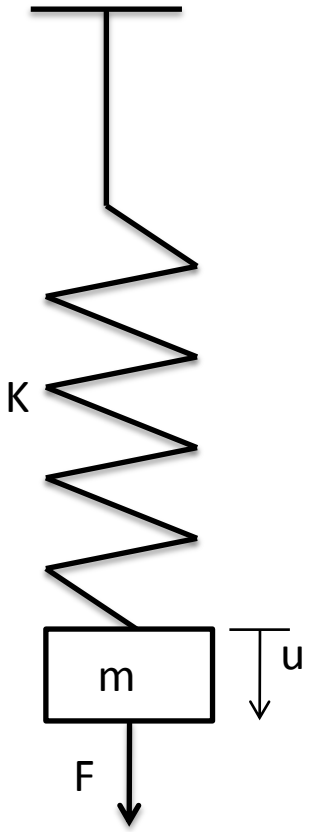


$$0 = \mathrm{m}\ddot{u} + ku$$

$$u = u\,\sin(\omega t)$$

Fjeder/masse system

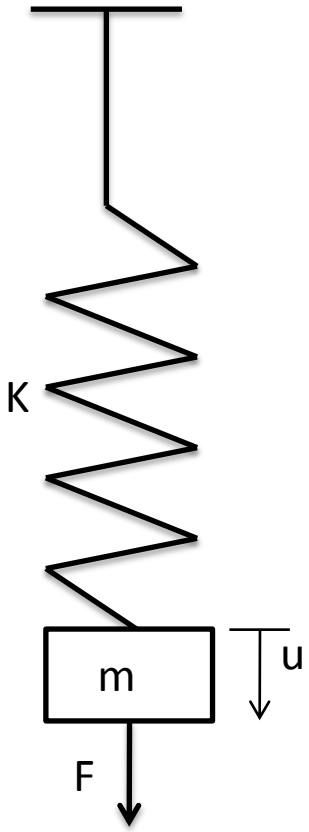


$$0 = \mathrm{m}\ddot{u} + ku$$

$$u = u\sin(\omega t)$$

$$\ddot{u} = -u\omega^2\sin(\omega t)$$

Fjeder/masse system



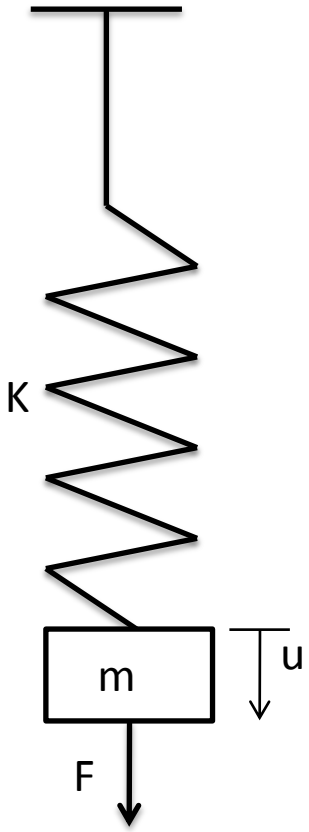
$$0 = \mathrm{m}\ddot{u} + ku$$

$$u = u\sin(\omega t)$$

$$\ddot{u} = -u\omega^2\sin(\omega t)$$

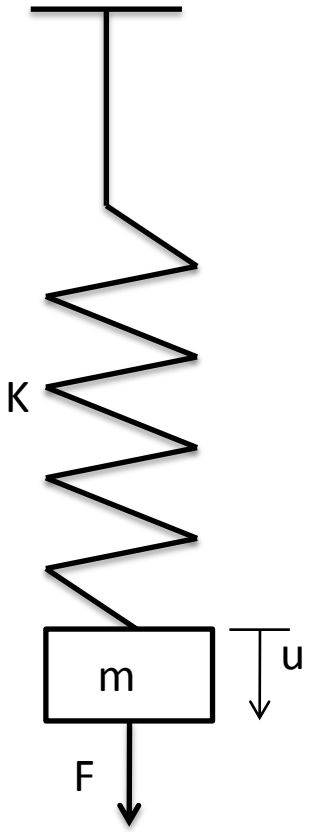
$$0 = -mu\omega^2\sin(\omega t) + ku\sin(\omega t)$$

Fjeder/masse system



$$0 = -m\ddot{u}\omega^2 \sin(\omega t) + k\ddot{u} \sin(\omega t)$$

Fjeder/masse system



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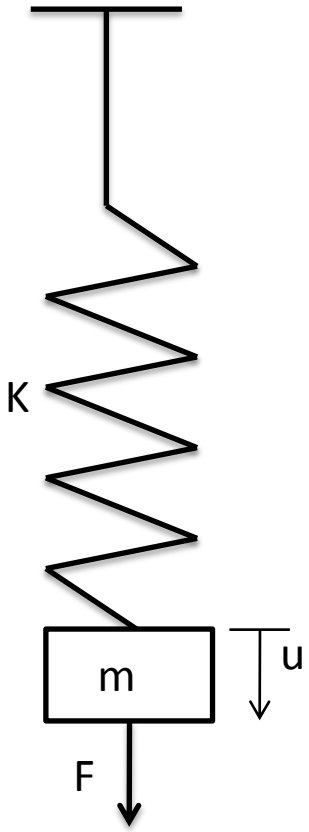
$$0 = -m\omega^2 + k$$

Fjeder/masse system

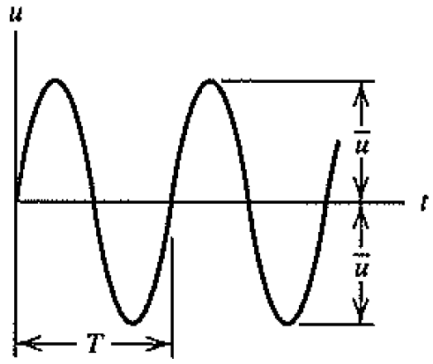


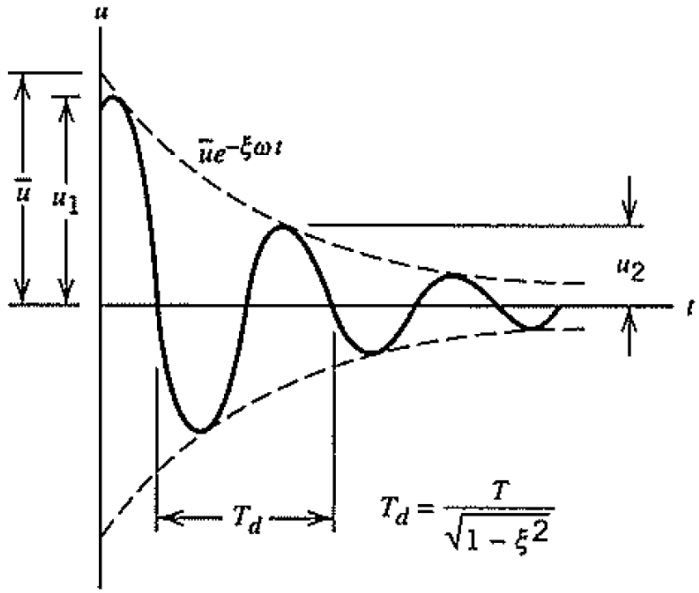
$$0 = -m\ddot{u}\omega^2 \sin(\omega t) + k\ddot{u} \sin(\omega t)$$

$$0 = -m\omega^2 + k$$

$$\omega = \sqrt{\frac{k}{m}}$$

Dæmpning


u
t
$\bar{u}$
$\bar{u}$
T


u
$\bar{u}$
u_1
$\bar{u}e^{-\xi\omega t}$
u_2
t
T_d
$T_d = \frac{T}{\sqrt{1-\xi^2}}$

Dæmpning

$$c \neq 0$$

$$C_{cr} = 2m\sqrt{\frac{k}{m}}$$

$$c < c_{cr}$$

Dæmpning

$$c \neq 0$$

$$C_{cr} = 2m\sqrt{\frac{k}{m}}$$

$$c < c_{cr}$$

$$\omega_d = \omega\sqrt{1-\xi^2} \qquad \xi = \frac{C}{C_{cr}}$$

Dæmpning

$$c \neq 0$$

$$\xi < 0{,}15$$

Dæmpning

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$$\omega_d = \omega\sqrt{1-\xi^2}$$

Dæmpning

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$$\omega_d = \omega\sqrt{1-\xi^2}$$

$$\omega_d = 30\sqrt{1-0{,}15^2} = 29{,}66\, rad/s$$

Dæmpning

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$$\omega_d = 30\sqrt{1-0{,}15^2} = 29{,}66\, rad/s$$

$$\omega_d = \frac{30}{2\pi} = 4{,}77Hz$$

$$\omega = \frac{29{,}66}{2\pi} = 4{,}72Hz$$

Dæmpning

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$$\omega_d \approx \omega$$

Matrise system

$$\{D\} = \{\overline{D}\} \sin(\omega t)$$

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$$0 = [\mathrm{m}]\{\ddot{D}\} + [k]\{D\}$$

Matrise system

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$$0 = -[\mathrm{m}]\{\overline{D}\}\omega^2 \sin(\omega t) + [k]\{\overline{D}\}\sin(\omega t)$$

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$$0 = [\mathrm{m}]\{\ddot{D}\} + [k]\{D\}$$

$$0 = -[\mathrm{m}]\{\overline{D}\}\omega^2 \cancel{\sin(\omega t)} + [k]\{\overline{D}\}\cancel{\sin(\omega t)}$$

Matrise system

$$\{D\} = \{\overline{D}\}\sin(\omega t)$$

$$\{\ddot{D}\} = -\{\overline{D}\}\omega^2 \sin(\omega t)$$

$$0 = [\mathrm{m}]\{\ddot{D}\} + [k]\{D\}$$

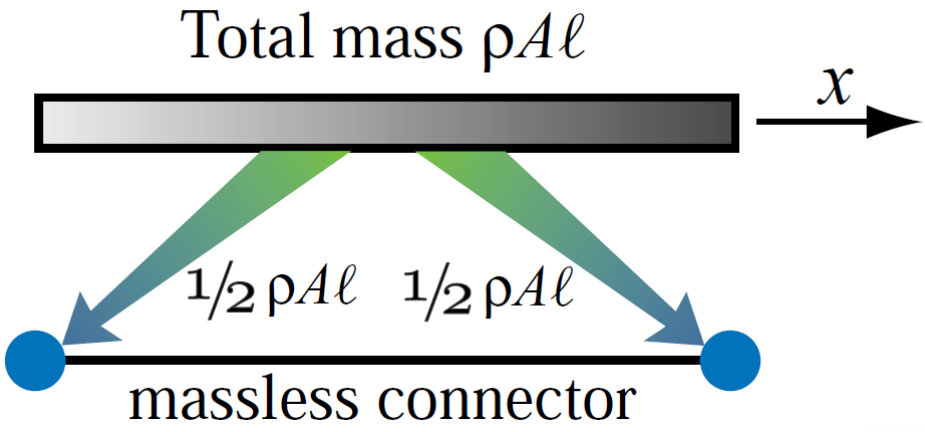
$$0 = -[\mathrm{m}]\{\overline{D}\}\omega^2 \sin(\omega t) + [k]\{\overline{D}\}\sin(\omega t)$$

$$0 = -[\mathrm{m}]\{\overline{D}\}\omega^2 + [k]\{\overline{D}\} \Rightarrow 0 = (-[\mathrm{m}]\omega^2 + [k])\{\overline{D}\}$$

Mass

Mass lumping

$$[\mathrm{m}] = \frac{m}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



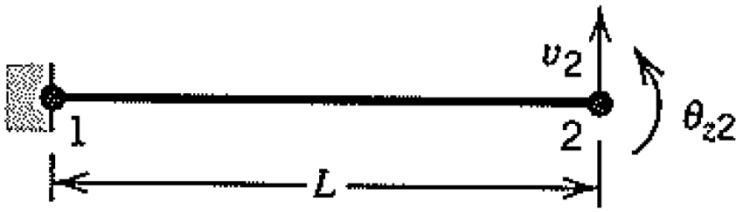
Mass

Consistent mass metrix

Beam:

$$\int_0^L [N]^T [N] \rho A \, dx = \frac{m}{420} \begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix}$$

Eksempel

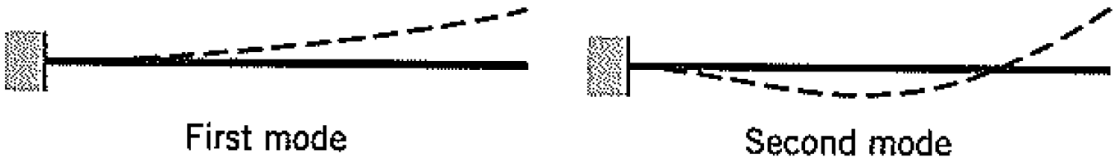


$$\left(\frac{EI}{L^3}\begin{bmatrix}12 & -6L\\ -6L & -4L^2\end{bmatrix} - \omega^2\frac{m}{420}\begin{bmatrix}156 & -22L\\ -22L & -4L^2\end{bmatrix}\right)\begin{Bmatrix}v_2\\ \theta_2\end{Bmatrix} = \begin{Bmatrix}0\\ 0\end{Bmatrix}$$

$$\omega_1 = 3{,}533\left(\frac{EI}{mL^3}\right)^{1/2}$$

$$\omega_2 = 34{,}81\left(\frac{EI}{mL^3}\right)^{1/2}$$

Eksempel



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