Fracture Mechanics - Examples

- Fracture mechanics can be applied in a large number of fields.
- The precise location of cracks are unpredictable
- The load level are more precisely estimated.

Breaking of Icebergs
Earthquakes

Different fracture modes:
- Opening mode
- Sliding mode
- Tearing mode
Opening mode – Mode I
• Involves normal stresses (tension)
• Normally the critical mode

Opening mode – Mode II and III
• Involves shearing stresses in- or out-of-plane
• Combination with mode I is often seen.

Some basic continuum mechanics
• Stresses
  • Plane stress, plane strain
• Strains
• Hooke’s law
• Elastic energy
• Polar coordinates
Singularity

Black board

Conclusion of continuum exercise

• At sharp edges there is a stress singularity
• The stresses go to infinity
• The strain energy density is limited
• The magnitude of the stress singularity determines the strain energy density or the stress intensity factor $K$
• Stress intensity factors should not be confused with stress concentration factors.
Stress concentration factors (SCF)

Changes in geometry will lead to a non-homogeneous stress state.

The ratio between the maximum stress and the uniform stress defines the stress concentration factor, SCF.

SCF in shaft

Parameters:
D = 50mm  d = 25mm  r = 2.5mm
D/d = 2  r/d = 0.1 gives SCF = 2
P = 20kN gives stress of 40.75 Mpa

Maximum stress = SCF \times 40.75\text{MPa} = 81.5\text{MPa}
Stress Intensity Factor for a small crack with a length of a:

\[ K = S \sqrt{a} \approx 1.77s\sqrt{a} \]

Parameters:
- \( S \): Normal stress
- \( a \): Crack length

For a very small crack:
\[ K = 1.99s\sqrt{a} \]
Analytical and semi-empirical solution

- In the literature there exist a number of solutions to some standard cases.
- Problem
  - Geometry are very restricted
  - Load cases simple
  - Validity best for very small cracks.
  - Mathematical complicated.
- Numerical methods i.e. FEA
  - Very general
  - Easy to operate

Exercise

Make a Finite Element analysis of a plate with a crack in the center.
Make the analysis for both plane strain and plane stress.
Plot the contours of the von Mises stress.
Compare the plane stress and plane strain situation.

NB: Exercise without units. Make your own choice.
Crack has to be relative small compared to other dimensions.
The mesh rather fine.
Finite Element .... to come

- J-integral for calculation of the Stress Intensity Factor
- Special elements around crack tip.
  - Included the squareroot singularity
- 2D-examples
- J-integral in 3D
- 3D-example